## THE ALYMPIC GAMES



Final assignment for the $\mathbf{2 4}{ }^{\text {rd }}$ Mathematics A-lympiade

Garderen, March 8 and 9, 2013


游 Texas Instruments

## Colophon

The Mathematics Alympiad (Wiskunde Alympiade) is an initiative of the Freudenthal Institute, Utrecht University

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## GUIDE FINAL MATHEMATICS A-LYMPIAD 2013

## IN ADVANCE:

- First read the full text of the assignment so you will know what you have to do this weekend
- Check whether you can open the files on the memory stick. The following files are on the stick:
- the assignment: Alympiade2012_2013 (in Dutch, English and German)
- the Excel files: DecathlonTable and DecathlonResults
- Divide tasks where possible.


## TIME MANAGEMENT:

- Keep an eye on the time when you work on the different parts.
- A report has to be handed in and a presentation has to be prepared. Plan in advance who works when on what.
- Saturday afternoon, before lunch, at 13.00 h , you must hand in the memory stick.


## RESEARCH QUESTION:

In several questions, it says: 'investigate how...'
Always indicate clearly what you investigated in these questions, if necessary investigate simpler problems, do more than simply answering the question, look into alternatives. In short, a research question is never 'done': every result can evoke new questions. You yourself determine how far you want to take your research.

## HANDING IN:

Saturday afternoon, at 13.00 h , you hand in the memory stick
Make sure the report is only one file, preferably a pdf. Test whether the pdf works on several different computers. The jury will receive digital copies of your report, so you can use colour. Each team is responsible for whether the file that it hands in can be opened and read.

## JUDGING:

Some of the points that are important for the jury:

- The elaboration and report of assignment 1 to 6 and the two final assignments
- The questions you ask yourself for the three research questions
- How complete the answers for the various parts are;
- The use of math;
- The argumentation used and justifying choices that have been made;
- The depth to which the various assignments have been answered;
- Presentation: shape, coherence, legibility, illustrations, etcetera;
- Originality and creativity.


## The Alympic Games

## Introduction

One of the most prestigious events at the Olympic Games is the decathlon. The winner is said to be the most versatile athlete: he scores highest on the combination of the ten different contests in it. Those are the running, throwing and jumping contests.
But... how do you combine these different times and distances to make one mutually comparable result? And how good were the winners from a hundred years ago compared with today's winners? That is the topic of this Alympiad assignment. You begin with investigating how the decathlon scoring system is set up, both today and historically. You will then look for a way to compare past and present performances.
After, you will design your own scoring system for the Alympic Tetrathlon.


## The decathlon

Traditionally, the title of "World's Greatest Athlete" has been given to the man who wins the Olympic decathlon. This began when King Gustav V of Sweden told Jim Thorpe, "You, sir, are the world's greatest athlete" after Thorpe won the decathlon at the Stockholm Olympics in 1912. The current decathlon world record holder is American Ashton Eaton, who scored 9,039 points at the 2012 United States Olympic Trials.

The decathlon is a sporting competition in which athletes have to take part in ten athletics events in two days. The decathlon has been a part of the Olympic Games since the 1912 Oslo Summer Olympics. The different parts of the decathlon are:

- 100 metres dash
- long jump
- shot put
- high jump
- 400 metres dash
- 110 metres hurdles
- discus throw
- pole vault
- javelin throw
- 1500 metres run

The athletes earn points for each event, according to a system laid down by the IAAF (International Association of Athletics Federations). The better the performance, the more points it yields. The athlete who has the most points after ten events, wins the decathlon. The current world record is held by the American Ashton Eaton with 9039 points.

Comparing a 100 metres dash with a high jump result is like comparing apples and oranges, but the decathlon scoring attempts to express distances and times in points as well as possible. The first scoring systems were rank-based: the order of each event was taken as the amount of points. The points were added, and the athlete with the lowest number of points was the winner. This system only takes the order per event into account, not the different performances. It doesn't make a difference whether someone wins by a hundredth of a second or by a mile: first place is one point.
The need for a more precise system, one that did take the different performances into account, soon became apparent.
You can find the first official scoring system from 1912 in the Excel file DecathlonTable.

## Assignment 1

Jim Thorpe, who was mentioned earlier, is seen by many as the greatest athlete of all time. His results on the decathlon at the 1912 Olympic Games were impressive.

| 100 metres dash | $11,2 \mathrm{~s}$ |
| :--- | ---: |
| long jump | $6,79 \mathrm{~m}$ |
| shot put | $12,89 \mathrm{~m}$ |
| high jump | $1,87 \mathrm{~m}$ |
| 400 metres dash | $52,2 \mathrm{~s}$ |
| 110 metres hurdles | $15,6 \mathrm{~s}$ |
| discus throw | $36,98 \mathrm{~m}$ |
| pole vault | $3,25 \mathrm{~m}$ |
| javelin throw | $45,7 \mathrm{~m}$ |
| 1500 metres run | $4: 40,1 \mathrm{~min}: \mathrm{s}$ |

Calculate Jim Thorpe's overall score at the 1912 Olympic Games. Use the Excel file DecathlonTable.

## Assignment 2

Investigate how the points for the various decathlon events were allocated in 1912. Show your approach and your findings (for example: similarities, differences, and remarkable findings) on one A4 sheet of paper maximum.
(There is an explanation of the research questions in the Guide)
Later scoring systems used other relations than a linear one. These are either progressive or regressive. A progressive system allocates significantly more points for a better result on an event, the better the result is. For instance: in the high jump, an improvement from $1,84 \mathrm{~m}$ to $1,85 \mathrm{~m}$ gives more extra points than an improvement from $1,74 \mathrm{~m}$ to $1,75 \mathrm{~m}$. A regressive system on the other hand will give fewer points for a better performance. Below you see a graphic representation of the three systems:


In 1934 a new scoring system was introduced, which was used for the first time at the 1936 Berlin Olympics. You will find this score in the Excel file DecathlonTable as well.

## Assignment 3

Calculate Jim Thorpe's overall score if he had taken part in 1936 and achieved the same results as in 1912.

## Assignment 4

Investigate how the points for the different events in the decathlon were assigned in 1934. Emphasise the differences with 1912 in your elaboration. At the very least look at whether the system is linear, progressive or regressive. Indicate what this means for athletes who excel at one event in comparison to those whose performance is more balanced.

The winner of the 1936 Olympic decathlon was the American Glenn Morris with the following results:

| 100 metres dash | $11,1 \mathrm{~s}$ |
| :--- | ---: |
| long jump | $6,97 \mathrm{~m}$ |
| shot put | $14,10 \mathrm{~m}$ |
| high jump | $1,85 \mathrm{~m}$ |
| 400 metres dash | $49,4 \mathrm{~s}$ |
| 110 metres hurdles | $14,9 \mathrm{~s}$ |
| discus throw | $43,02 \mathrm{~m}$ |
| pole vault | $3,50 \mathrm{~m}$ |
| javelin throw | $54,52 \mathrm{~m}$ |
| 1500 metres run | $4: 33,2 \mathrm{~min}: \mathrm{s}$ |

## Assignment 5

Investigate, using the 1934 scoring system, on which decathlon events Glenn Morris will profit most if he improves his performance on that event.

The IAAF wants a scoring system that meets the following two requirements:

- Every event contributes roughly the same amount of points to the overall score,
- A good score on a single event may not be decisive for the final result.

To meet these requirements a new system was introduced in 1984. This current system calculates the score as follows:

- Running events: score $=a(b-T)^{c}$ in which $T$ represents the time in seconds
- Jumping events: score $=a(M-b)^{c}$ in which $M$ represents the distance jumped in centimetres
- Throwing events: score $=a(D-b)^{c}$ in which $D$ represents the throwing distance in meters
$a, b$ and $c$ are parameters that differ per event, as can be seen in the table below:

| Event | $\boldsymbol{a}$ | $\boldsymbol{b}$ | $\boldsymbol{c}$ |
| :--- | :---: | :---: | :---: |
| 100 metres dash | 25,4347 | 18 | 1,81 |
| long jump | 0,14354 | 220 | 1,4 |
| shot put | 51,39 | 1,5 | 1,05 |
| high jump | 0,8465 | 75 | 1,42 |
| 400 metres dash | 1,53775 | 82 | 1,81 |
| 110 metres hurdles | 5,74352 | 28,5 | 1,92 |
| discus throw | 12,91 | 4 | 1,1 |
| pole vault | 0,2797 | 100 | 1,35 |
| javelin throw | 10,14 | 7 | 1,08 |
| 1500 metres run | 0,03768 | 480 | 1,85 |

The result of the calculation is rounded off downwards to an integer.
The current world champion and world record holder on the decathlon is the American Ashton Eaton. These are the results from the 2012 London Olympics that led to his points world record:

| 100 metres dash | $10,21 \mathrm{~s}$ |
| :--- | ---: |
| long jump | $8,23 \mathrm{~m}$ |
| shot put | $14,2 \mathrm{~m}$ |
| high jump | $2,05 \mathrm{~m}$ |
| 400 metres dash | $46,7 \mathrm{~s}$ |
| 110 metres hurdles | $12,7 \mathrm{~s}$ |
| discus throw | $42,81 \mathrm{~m}$ |
| pole vault | $5,3 \mathrm{~m}$ |
| javelin throw | $58,87 \mathrm{~m}$ |
| 1500 metres run | $04: 14,5 \mathrm{~min}: \mathrm{s}$ |

## Assignment 6

Calculate Ashton Eaton's points world record.
How many points would Jim Thorpe's results from 1934 yield in the current system?
It is obvious that Ashton Eaton scores higher on all events than Jim Thorpe. But the two athletes live in different times. There have been many changes in materials, circumstances, training facilities and many other factors over the last century. Even so, we still want to know who is the greatest athlete of all time. Or, more generally, how do we compare the results from a modern athlete with those from the past?
One way to do that is to use a measure that represents performances in specific years
objectively. And that measure exists: for every year, the world record for every event in the decathlon is known. You will find the world records in the Excel File Worldrecords.

## Final assignment 1

Develop a scoring system that allows for the best comparison of the results of decathletes from different periods in time by relating the results to world records. Then use your system to rank the following decathlon results: who is the best decathlete of all time according to your group? You will also find these results in the Excel file DecathlonResults.

| Name | Year | $\mathbf{1 0 0} \mathbf{m}$ | long | shot | high | $\mathbf{4 0 0} \mathbf{m}$ | $\mathbf{1 1 0} \mathbf{m} \mathbf{h}$ | discus | pole | javelin | $\mathbf{1 5 0 0} \mathbf{m}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Jim Thorpe | $\mathbf{1 9 1 2}$ | 11,2 | 6,79 | 12,89 | 1,87 | 52,2 | 15,6 | 36,98 | 3,25 | 45,7 | $4: 40,1$ |
| Helge Løvland | $\mathbf{1 9 2 0}$ | 12,0 | 8,28 | 11,19 | 1,65 | 54,8 | 16,2 | 37,34 | 3,20 | 48,01 | $4: 48,4$ |
| Paavo Yrjölä | $\mathbf{1 9 2 8}$ | 11,8 | 6,72 | 14,11 | 1,87 | 53,2 | 16,6 | 42,09 | 3,30 | 55,7 | $4: 44,0$ |
| Glenn Morris | $\mathbf{1 9 3 6}$ | 11,1 | 6,97 | 14,10 | 1,85 | 49,4 | 14,9 | 43,02 | 3,50 | 54,52 | $4: 33,2$ |
| Bob Mathias | $\mathbf{1 9 5 0}$ | 10,9 | 7,09 | 14,48 | 1,85 | 51,0 | 14,7 | 44,62 | 3,98 | 55,59 | $5: 05,1$ |
| Rafer Johnson | $\mathbf{1 9 6 0}$ | 10,9 | 7,35 | 15,82 | 1,85 | 48,3 | 15,3 | 48,49 | 4,10 | 69,76 | $4: 49,1$ |
| Russel Hodge | $\mathbf{1 9 6 6}$ | 10,5 | 1,51 | 17,25 | 1,85 | 48,9 | 15,2 | 50,44 | 4,10 | 64,49 | $4: 40,4$ |
| Bruce Jenner | $\mathbf{1 9 7 6}$ | 10,94 | 7,22 | 15,35 | 2,03 | 47,51 | 14,84 | 50,04 | 4,80 | 68,52 | $4: 12,61$ |
| Siegfried Stark | $\mathbf{1 9 8 0}$ | 11,10 | 7,64 | 15,81 | 2,03 | 49,53 | 14,86 | 47,2 | 5,00 | 68,70 | $4: 27,70$ |
| Daley Thompson | $\mathbf{1 9 8 4}$ | 10,44 | 8,01 | 15,72 | 2,03 | 46,97 | 14,33 | 46,56 | 5,00 | 65,24 | $4: 35,00$ |
| Dan O'Brien | $\mathbf{1 9 9 2}$ | 10,43 | 8,08 | 16,69 | 2,07 | 48,51 | 13,98 | 48,56 | 5,00 | 62,58 | $4: 42,10$ |
| Tomáš Dvořák | $\mathbf{1 9 9 9}$ | 10,54 | 7,90 | 16,78 | 2,04 | 48,08 | 13,73 | 48,33 | 4,90 | 72,32 | $4: 37,20$ |
| Roman Šebrle | $\mathbf{2 0 0 1}$ | 10,64 | 8,11 | 15,33 | 2,12 | 47,79 | 13,92 | 47,92 | 4,80 | 70,16 | $4: 21,98$ |
| Bryan Clay | $\mathbf{2 0 0 8}$ | 10,39 | 7,39 | 15,17 | 2,08 | 48,41 | 13,75 | 52,74 | 5,00 | 70,55 | $4: 50,97$ |
| Eelco Sintnicolaas | $\mathbf{2 0 1 2}$ | 10,77 | 7,27 | 14,20 | 2,00 | 48,02 | 14,10 | 42,81 | 5,36 | 63,59 | $4: 26,98$ |
| Ashton Eaton | $\mathbf{2 0 1 2}$ | 10,21 | 8,23 | 14,20 | 2,05 | 46,70 | 13,70 | 42,81 | 5,30 | 58,87 | $4: 14,48$ |

In the report for this assignment you will go into:

- The design of the 'ageless' scoring system
- The motivation for choices you made in the design
- Examples of calculations that support your 'all time ranking'


## Final assignment 2

During the final weekend your group took part in all four events in the Alympiad
Tetrathlon. You can find the results of all groups in the Excel file AlympiadTetrathlon.
You will be given this file on Saturday morning.
Design a scoring system for the Alympiad Tetrathlon.

## Presentation

The final weekend ends with a presentation on Saturday afternoon. More detailed information will be given on Friday afternoon.

